CONGRUENCE & SIMILARITY OF TRIANGLES

Complete SSC CGL Geometry Notes

Comprehensive Guide with Theorems & Examples • Created by GovtExamPrep

1. CONGRUENCE OF TRIANGLES - BASIC CONCEPTS

Definition and Meaning

Congruent Triangles: Two triangles are congruent if they have exactly the same size and shape. This means:
• Corresponding sides are equal
• Corresponding angles are equal
• They can be superimposed exactly on each other

[Diagram: Two congruent triangles showing corresponding sides and angles]

Symbol: ≅
Notation: If $\Delta ABC \cong \Delta PQR$, then:
• $AB = PQ$, $BC = QR$, $CA = RP$
• $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

Criteria for Congruence

Criterion	Condition	Abbreviation
Side-Side-Side	All three sides equal	SSS
Side-Angle-Side	Two sides and included angle equal	SAS
Angle-Side-Angle	Two angles and included side equal	ASA
Angle-Angle-Side	Two angles and non-included side equal	AAS
Right-Hypotenuse-Side	Right angle, hypotenuse and one side equal	RHS

Important: AAA (Angle-Angle-Angle) is NOT a congruence criterion. It only ensures similarity, not congruence.

2. CONGRUENCE CRITERIA - DETAILED EXPLANATION

SSS (Side-Side-Side) Congruence

Theorem: If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent.

Example: In triangles ABC and PQR, AB = PQ = 5cm, BC = QR = 6cm, AC = PR = 7cm. Prove $\Delta ABC \cong \Delta PQR$

Solution:

- AB = PQ = 5cm (Given)
- BC = QR = 6cm (Given)
- AC = PR = 7cm (Given)
- Therefore, by SSS criterion, $\Delta ABC \cong \Delta PQR$

SAS (Side-Angle-Side) Congruence

Theorem: If two sides and the included angle of one triangle are equal to two sides and included angle of another triangle, then the triangles are congruent.

Example: In ΔABC and ΔDEF, AB = DE = 4cm, $\angle B = \angle E = 60°$, BC = EF = 5cm. Prove congruence.

Solution:

- AB = DE = 4cm (Given)
- $\angle B = \angle E = 60°$ (Given)
- BC = EF = 5cm (Given)
- Angle B is included between AB and BC
- Angle E is included between DE and EF
- Therefore, by SAS criterion, $\Delta ABC \cong \Delta DEF$

ASA (Angle-Side-Angle) Congruence

Theorem: If two angles and the included side of one triangle are equal to two angles and included side of another triangle, then the triangles are congruent.

Example: In ΔPQR and ΔXYZ, ∠P = ∠X = 50°, PQ = XY = 6cm, ∠Q = ∠Y = 70°. Prove congruence.

Solution:

- ∠P = ∠X = 50° (Given)
- PQ = XY = 6cm (Given)
- ∠Q = ∠Y = 70° (Given)
- Side PQ is included between ∠P and ∠Q
- Side XY is included between ∠X and ∠Y
- Therefore, by ASA criterion, ΔPQR ≅ ΔXYZ

3. SIMILARITY OF TRIANGLES - BASIC CONCEPTS

Definition and Meaning

Similar Triangles: Two triangles are similar if they have the same shape but not necessarily the same size. This means:
• Corresponding angles are equal
• Corresponding sides are proportional

[Diagram: Two similar triangles showing proportional sides and equal angles]

Symbol: ~
Notation: If ΔABC ~ ΔPQR, then:
• ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R
• AB/PQ = BC/QR = CA/RP = k (constant ratio)

Criteria for Similarity

Criterion	Condition	Abbreviation
Angle-Angle-Angle	All three angles equal	AAA
Side-Side-Side	All three sides proportional	SSS
Side-Angle-Side	Two sides proportional and included angle equal	SAS

Important: For similarity, AAA is sufficient (unlike congruence where AAA is not sufficient).

4. SIMILARITY CRITERIA - DETAILED EXPLANATION

AAA (Angle-Angle-Angle) Similarity

Theorem: If in two triangles, corresponding angles are equal, then their corresponding sides are proportional and hence the triangles are similar.

Example: In ΔABC and ΔPQR, ∠A = ∠P = 40°, ∠B = ∠Q = 60°, ∠C = ∠R = 80°. Prove similarity.

Solution:

- ∠A = ∠P = 40° (Given)
- ∠B = ∠Q = 60° (Given)
- ∠C = ∠R = 80° (Given)
- All corresponding angles are equal
- Therefore, by AAA criterion, ΔABC ~ ΔPQR

SSS (Side-Side-Side) Similarity

Theorem: If in two triangles, sides of one triangle are proportional to the sides of another triangle, then their corresponding angles are equal and hence the triangles are similar.

Example: In ΔABC and ΔDEF, AB/DE = BC/EF = CA/FD = 2/3. Prove similarity.

Solution:

- AB/DE = 2/3 (Given)
- BC/EF = 2/3 (Given)
- CA/FD = 2/3 (Given)
- All corresponding sides are proportional
- Therefore, by SSS criterion, ΔABC ~ ΔDEF
- Scale factor = 2/3

SAS (Side-Angle-Side) Similarity

Theorem: If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, then the triangles are similar.

Example: In ΔPQR and ΔXYZ, ∠Q = ∠Y = 50°, PQ/XY = QR/YZ = 3/4. Prove similarity.

Solution:

- ∠Q = ∠Y = 50° (Given)
- PQ/XY = 3/4 (Given)
- QR/YZ = 3/4 (Given)
- Sides including equal angles are proportional
- Therefore, by SAS criterion, ΔPQR ~ ΔXYZ

5. COMPARISON: CONGRUENCE vs SIMILARITY

Key Differences

Aspect	Congruence	Similarity
Size	Same size	Different sizes possible
Shape	Same shape	Same shape
Symbol	≅	~
AAA Criterion	Not applicable	Applicable
Side Ratio	Exactly equal (1:1)	Proportional (k:1)

Relationship: All congruent triangles are similar (with scale factor 1), but all similar triangles are not necessarily congruent.

Scale Factor and Ratio

Scale Factor (k):
`k = (Side of triangle 1) / (Corresponding side of triangle 2)`

Area Ratio:
`(Area of triangle 1) / (Area of triangle 2) = k²`

For congruent triangles: `k = 1, so areas are equal`

Example: If ΔABC ~ ΔPQR with scale factor 3/2, and area of ΔABC = 36cm^2, find area of ΔPQR.

Solution:

- Scale factor k = 3/2
- Area ratio = $k^2 = (3/2)^2 = 9/4$
- Area(ABC)/Area(PQR) = 9/4

- 36/Area(PQR) = 9/4
- Area(PQR) = (36 × 4)/9 = 16cm^2

6. IMPORTANT THEOREMS AND PROPERTIES

Basic Proportionality Theorem (Thales Theorem)

Theorem: If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

Example: In ΔABC, DE ∥ BC. If AD = 3cm, DB = 2cm, AE = 4.5cm, find EC.

Solution:

- By Basic Proportionality Theorem:
- AD/DB = AE/EC
- 3/2 = 4.5/EC
- EC = (4.5 × 2)/3 = 3cm

Pythagoras Theorem

Theorem: In a right-angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.
$AC^2 = AB^2 + BC^2$ (where ∠B = 90°)

Example: In right ΔABC, ∠B = 90°, AB = 6cm, BC = 8cm. Find AC.

Solution:

- By Pythagoras Theorem: $AC^2 = AB^2 + BC^2$
- $AC^2 = 6^2 + 8^2 = 36 + 64 = 100$
- AC = √100 = 10cm

7. SSC CGL PRACTICE PROBLEMS

Previous Year Question Types

Problem 1: In ΔABC and ΔDEF, AB = DE, ∠A = ∠D, ∠B = ∠E. Which congruence criterion applies?

Solution:

- AB = DE (Given)
- ∠A = ∠D (Given)
- ∠B = ∠E (Given)
- Side AB is included between ∠A and ∠B
- Therefore, **ASA criterion** applies

Problem 2: ΔABC ~ ΔPQR with AB/PQ = 3/5. If BC = 12cm, find QR.

Solution:

- Scale factor k = AB/PQ = 3/5
- BC/QR = 3/5
- 12/QR = 3/5
- QR = (12 × 5)/3 = 20cm

Problem 3: In ΔABC, DE ∥ BC, AD = 4cm, BD = 6cm, AE = 5cm. Find AC.

Solution:

- By Basic Proportionality Theorem:
- AD/BD = AE/EC
- 4/6 = 5/EC
- EC = (5 × 6)/4 = 7.5cm
- AC = AE + EC = 5 + 7.5 = 12.5cm

8. QUICK FORMULAS & SHORTCUTS

Memory Tips

Congruence Criteria:
• SSS - All sides equal
• SAS - Two sides and included angle equal
• ASA - Two angles and included side equal
• AAS - Two angles and any side equal
• RHS - Right angle, hypotenuse, side

Similarity Criteria:
• AAA - All angles equal
• SSS - All sides proportional
• SAS - Two sides proportional and included angle equal

Area Relationships:
• For similar triangles: Area ratio = $(\text{Scale factor})^2$
• For congruent triangles: Areas equal

Important Values

Scale Factor (k)	Side Ratio	Area Ratio	Perimeter Ratio
1/2	1:2	1:4	1:2
2/3	2:3	4:9	2:3
3/4	3:4	9:16	3:4
1 (Congruent)	1:1	1:1	1:1

9. SSC CGL PREPARATION STRATEGY

Expected Marks Distribution

Topic	Frequency	Difficulty	Marks Weightage
Congruence Criteria	High	Easy-Medium	2-3 marks
Similarity Criteria	High	Medium	2-3 marks
Basic Proportionality	Medium	Medium	1-2 marks
Area Problems	Medium	Medium-Hard	1-2 marks
Mixed Problems	High	Hard	2-3 marks

25-Day Study Plan

Week 1-2: Foundation Building
• Days 1-5: Congruence concepts and criteria
• Days 6-10: Similarity concepts and criteria
• Days 11-15: Comparison and applications

Week 3-4: Advanced Practice
• Days 16-20: Theorems and proofs
• Days 21-25: Previous year papers and mock tests

Daily Practice: 5 congruence problems, 5 similarity problems, learn all criteria, practice area calculations.

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