TRIANGLES & THEIR CENTERS

Complete SSC CGL Master Notes

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1. TRIANGLE FUNDAMENTALS & TYPES

Basic Definition

A **triangle** is a polygon with three edges and three vertices. It is one of the basic shapes in geometry.

Triangle ABC with:
• Vertices: A, B, C
• Sides: AB, BC, CA
• Angles: ∠A, ∠B, ∠C
• Sum of angles: ∠A + ∠B + ∠C = 180°

Types of Triangles

Based on Sides	Properties	Area Formula
Equilateral	All sides equal, all angles 60°	$(\sqrt{3}/4) \times a^2$
Isosceles	Two sides equal, two angles equal	$(b/4)\sqrt{(4a^2 - b^2)}$
Scalene	All sides different	$\sqrt{[s(s-a)(s-b)(s-c)]}$

Based on Angles	Properties	Area Formula
Acute	All angles < 90°	½ × base × height
Right	One angle = 90°	½ × base × height
Obtuse	One angle > 90°	½ × base × height

2. IMPORTANT TRIANGLE FORMULAS

Area Formulas

1. Basic Formula: `Area = ½ × base × height`
2. Heron's Formula: `Area = √[s(s-a)(s-b)(s-c)] where s = (a+b+c)/2`
3. Using Trigonometry: `Area = ½ × ab × sin(C)`
4. Equilateral Triangle: `Area = (√3/4) × a²`
5. Right Triangle: `Area = ½ × (leg1 × leg2)`

Perimeter & Semi-perimeter

Perimeter: `P = a + b + c`
Semi-perimeter: `s = (a + b + c)/2`
For Equilateral: `P = 3a, s = 3a/2`

3. CENTROID - CENTER OF GRAVITY

Definition & Properties

Centroid (G) is the point where all three medians of the triangle intersect.
It is the center of gravity of the triangle.

Coordinates: $G = [(x_1+x_2+x_3)/3, (y_1+y_2+y_3)/3]$
Property 1: Divides each median in ratio 2:1
Property 2: Always lies inside the triangle
Property 3: Centroid of area = Centroid of vertices

Medians & Centroid Theorem

Theorem: Centroid divides median in 2:1 ratio

Proof:

- In triangle ABC, AD is median
- Centroid G lies on AD
- AG : GD = 2 : 1
- Similarly for other medians

Example: Triangle with vertices (0,0), (4,0), (0,3). Find centroid.

Solution:

- G = [(0+4+0)/3, (0+0+3)/3]
- G = [4/3, 3/3] = [4/3, 1]
- **Centroid: (1.33, 1)**

4. INCENTER - CENTER OF INCIRCLE

Definition & Properties

Incenter (I) is the point where all three angle bisectors of the triangle intersect. It is the center of the inscribed circle (incircle).

Coordinates: $I = [(ax_1+bx_2+cx_3)/(a+b+c), (ay_1+by_2+cy_3)/(a+b+c)]$
Property 1: Equidistant from all three sides
Property 2: Always lies inside the triangle
Property 3: Inradius $r = \Delta/s$ where Δ = area, s = semi-perimeter

Angle Bisector Theorem

Angle Bisector Theorem:
If AD is angle bisector of $\angle A$ in ΔABC, then:
$BD/DC = AB/AC$

Example: In triangle with sides 5,6,7, find inradius

Solution:

- $s = (5+6+7)/2 = 9$
- Area $= \sqrt{[9(9-5)(9-6)(9-7)]} = \sqrt{[9\times4\times3\times2]} = \sqrt{216} = 6\sqrt{6}$
- Inradius $r = \Delta/s = (6\sqrt{6})/9 = (2\sqrt{6})/3$
- **Inradius: $2\sqrt{6}/3 \approx 1.633$**

5. CIRCUMCENTER - CENTER OF CIRCUMCIRCLE

Definition & Properties

Circumcenter (O) is the point where all three perpendicular bisectors of the sides intersect.
It is the center of the circumscribed circle (circumcircle).

Property 1: Equidistant from all three vertices
Property 2: Position depends on triangle type:
- Acute: Inside triangle
- Right: On hypotenuse midpoint
- Obtuse: Outside triangle

Property 3: Circumradius R = abc/4Δ

Circumradius Formulas

General Triangle: R = abc/4Δ
Right Triangle: R = hypotenuse/2
Equilateral Triangle: R = a/√3
Using sides & angles: R = a/(2sinA) = b/(2sinB) = c/(2sinC)

Example: Right triangle with sides 3,4,5. Find circumradius.

Solution:

- For right triangle, R = hypotenuse/2
- Hypotenuse = 5
- R = 5/2 = **2.5**

6. ORTHOCENTER - INTERSECTION OF ALTITUDES

Definition & Properties

Orthocenter (H) is the point where all three altitudes of the triangle intersect.

Property 1: `Position depends on triangle type:`
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  - Acute: Inside triangle
  - Right: At right angle vertex
  - Obtuse: Outside triangle
```
Property 2: `In acute triangle, orthocenter lies inside`
Property 3: `In right triangle, orthocenter = right angle vertex`

Euler Line Theorem

Euler Line Theorem:
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In any triangle, Centroid (G), Circumcenter (O), and
Orthocenter (H) are collinear.
G divides OH in ratio 2:1 (OG:GH = 2:1)
O---G---H (OG = 2×GH)
```

Example: Equilateral triangle - Special case

Solution:

- In equilateral triangle, all four centers coincide
- Centroid = Incenter = Circumcenter = Orthocenter
- They all lie at the same point

7. COMPARISON OF ALL CENTERS

Centers Summary Table

Center	Definition	Properties	Position
Centroid (G)	Intersection of medians	Divides medians 2:1, Center of gravity	Always inside
Incenter (I)	Intersection of angle bisectors	Equidistant from sides, Incircle center	Always inside
Circumcenter (O)	Intersection of perpendicular bisectors	Equidistant from vertices, Circumcircle center	Inside/On/Outside
Orthocenter (H)	Intersection of altitudes	-	Inside/On/Outside

Special Cases

Equilateral Triangle:
• All four centers coincide at the same point
• This point is the center of both incircle and circumcircle

Isosceles Triangle:
• All centers lie on the axis of symmetry
• Centroid, Incenter, Circumcenter, Orthocenter are collinear

Right Triangle:
• Circumcenter = midpoint of hypotenuse
• Orthocenter = right angle vertex

8. SSC CGL PRACTICE PROBLEMS

Problem Set with Solutions

Problem 1: Find centroid of triangle with vertices (1,2), (3,4), (5,6)

Solution:

- G = [(1+3+5)/3, (2+4+6)/3]
- G = [9/3, 12/3] = [3, 4]
- **Centroid: (3,4)**

Problem 2: Equilateral triangle side 6cm. Find inradius and circumradius.

Solution:

- Inradius r = $a/(2\sqrt{3}) = 6/(2\sqrt{3}) = 3/\sqrt{3} = \sqrt{3}$ cm
- Circumradius R = $a/\sqrt{3} = 6/\sqrt{3} = 2\sqrt{3}$ cm
- **r = $\sqrt{3}$ cm, R = $2\sqrt{3}$ cm**

Problem 3: Right triangle sides 6cm, 8cm, 10cm. Find distance between incenter and circumcenter.

Solution:

- For right triangle: O = hypotenuse midpoint = (5,0)
- Incenter coordinates: use formula
- Distance OI = $\sqrt{[(R-2r)^2]}$ = R - 2r (since R > 2r)
- R = 5, r = (a+b-c)/2 = (6+8-10)/2 = 2
- OI = 5 - 4 = **1 cm**

Important Theorems for SSC CGL

1. **Apollonius Theorem:** $AB^2 + AC^2 = 2(AD^2 + BD^2)$ where AD is median
2. **Angle Bisector Theorem:** BD/DC = AB/AC
3. **Pythagoras Theorem:** $a^2 + b^2 = c^2$ (for right triangle)

4. Basic Proportionality Theorem: If DE ∥ BC, then AD/DB = AE/EC

5. Euler Line Theorem: O, G, H are collinear with OG:GH = 2:1

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