

SQUARE ROOTS

Complete SSC CGL Mathematics Notes

Comprehensive Guide with Methods & Examples • Created by GovtExamPrep

1. BASIC CONCEPTS OF SQUARE ROOTS

Definition and Notation

Square Root: The square root of a number is a value that, when multiplied by itself, gives the original number.

If $a^2 = b$, then a is the square root of b .

Notation: $\sqrt{b} = a$

Mathematical Representation:

If $a^2 = b$, then $\sqrt{b} = a$

Examples:

- $4^2 = 16$, so $\sqrt{16} = 4$
- $7^2 = 49$, so $\sqrt{49} = 7$
- $12^2 = 144$, so $\sqrt{144} = 12$

Number	Square	Square Root
1	1	1
2	4	$\sqrt{2} \approx 1.414$
3	9	$\sqrt{3} \approx 1.732$
4	16	2
5	25	$\sqrt{5} \approx 2.236$
6	36	$\sqrt{6} \approx 2.449$
7	49	$\sqrt{7} \approx 2.646$
8	64	$\sqrt{8} \approx 2.828$
9	81	3
10	100	$\sqrt{10} \approx 3.162$

Properties of Square Roots

Important Properties:

- $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{a/b} = \sqrt{a} / \sqrt{b}$ (where $b \neq 0$)
- $(\sqrt{a})^2 = a$
- $\sqrt{a^2} = |a|$ (absolute value)
- $\sqrt{a}$ is always non-negative for real numbers

Example: Verify properties with numbers

Solution:

- $\sqrt{4 \times 9} = \sqrt{36} = 6$ and $\sqrt{4} \times \sqrt{9} = 2 \times 3 = 6 \checkmark$
- $\sqrt{25/4} = \sqrt{6.25} = 2.5$ and $\sqrt{25}/\sqrt{4} = 5/2 = 2.5 \checkmark$
- $(\sqrt{16})^2 = 4^2 = 16 \checkmark$
- $\sqrt{5^2} = \sqrt{25} = 5 = |5| \checkmark$

2. PERFECT SQUARES AND THEIR ROOTS

Understanding Perfect Squares

Perfect Square: A number that is the square of an integer.

Examples: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, ...

Identifying Perfect Squares:

- Ends with 0, 1, 4, 5, 6, or 9
- Digital root is 1, 4, 7, or 9
- Even number of zeros at the end
- Last two digits form specific patterns

Number	Perfect Square?	Reason
144	Yes	$12^2 = 144$
200	No	$\sqrt{200} \approx 14.142$ (not integer)
625	Yes	$25^2 = 625$
1000	No	$\sqrt{1000} \approx 31.622$ (not integer)

Common Perfect Squares (1-30)

Perfect Squares Table:

$1^2=1$, $2^2=4$, $3^2=9$, $4^2=16$, $5^2=25$, $6^2=36$, $7^2=49$, $8^2=64$, $9^2=81$, $10^2=100$
 $11^2=121$, $12^2=144$, $13^2=169$, $14^2=196$, $15^2=225$, $16^2=256$, $17^2=289$, $18^2=324$,
 $19^2=361$, $20^2=400$
 $21^2=441$, $22^2=484$, $23^2=529$, $24^2=576$, $25^2=625$, $26^2=676$, $27^2=729$, $28^2=784$,
 $29^2=841$, $30^2=900$

Memory Tip: Remember these key perfect squares:

$12^2=144$, $13^2=169$, $14^2=196$, $15^2=225$, $16^2=256$, $17^2=289$, $18^2=324$, $19^2=361$,
 $20^2=400$

3. METHODS TO FIND SQUARE ROOTS

Prime Factorization Method

Steps for Prime Factorization:

1. Find prime factors of the number
2. Group the factors in pairs
3. Take one factor from each pair
4. Multiply these factors to get square root

Example: Find $\sqrt{1764}$ using prime factorization

Solution:

- Prime factors of 1764: $2 \times 2 \times 3 \times 3 \times 7 \times 7$
- Group in pairs: $(2 \times 2) \times (3 \times 3) \times (7 \times 7)$
- Take one from each pair: $2 \times 3 \times 7$
- $\sqrt{1764} = 2 \times 3 \times 7 = 42$
- Verification: $42^2 = 1764 \checkmark$

Example: Find $\sqrt{1296}$ using prime factorization

Solution:

- Prime factors of 1296: $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
- Group in pairs: $(2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (3 \times 3)$
- Take one from each pair: $2 \times 2 \times 3 \times 3$
- $\sqrt{1296} = 2 \times 2 \times 3 \times 3 = 36$

Long Division Method

Long Division Steps:

1. Group digits in pairs from right
2. Find largest number whose square $\leq$ first group
3. Subtract and bring down next pair
4. Double the quotient and find next digit
5. Repeat until all pairs are used

Example: Find $\sqrt{576}$ using long division**Solution:**

- Group: 5 76
- Largest number with square ≤ 5 is 2 ($2^2=4$)
- Subtract: $5-4=1$, bring down 76 $\rightarrow 176$
- Double quotient: $2 \times 2=4$, find digit x such that $4x \times x \leq 176$
- $44 \times 4 = 176$ exactly
- $\sqrt{576} = 24$

4. SQUARE ROOTS OF DECIMALS AND FRACTIONS

Decimal Square Roots

For Decimals:

- Make pairs from decimal point both ways
- Use long division method
- Place decimal point in quotient when you reach decimal point in dividend

Example: Find $\sqrt{2.25}$

Solution:

- $2.25 = 225/100$
- $\sqrt{(225/100)} = \sqrt{225}/\sqrt{100} = 15/10 = 1.5$
- Verification: $1.5^2 = 2.25 \checkmark$

Example: Find $\sqrt{0.09}$

Solution:

- $0.09 = 9/100$
- $\sqrt{(9/100)} = \sqrt{9}/\sqrt{100} = 3/10 = 0.3$
- Verification: $0.3^2 = 0.09 \checkmark$

Fraction Square Roots

For Fractions:

$$\sqrt{(a/b)} = \sqrt{a} / \sqrt{b}$$

For Mixed Fractions:

Convert to improper fraction first

Example: Find $\sqrt{(25/64)}$

Solution:

- $\sqrt{(25/64)} = \sqrt{25}/\sqrt{64} = 5/8$

- Verification: $(5/8)^2 = 25/64$ ✓

Example: Find $\sqrt{2\frac{1}{4}}$

Solution:

- $2\frac{1}{4} = 9/4$
- $\sqrt{9/4} = \sqrt{9}/\sqrt{4} = 3/2 = 1.5$
- Verification: $1.5^2 = 2.25 = 2\frac{1}{4}$ ✓

5. APPROXIMATION AND ESTIMATION

Estimation Method

Estimation Steps:

1. Find nearest perfect squares
2. Identify range
3. Use linear approximation if needed
4. Refine estimate

Example: Estimate $\sqrt{50}$

Solution:

- Nearest perfect squares: 49 (7^2) and 64 (8^2)
- 50 is closer to 49 than to 64
- Difference: $50 - 49 = 1$, Total difference: $64 - 49 = 15$
- Estimate: $7 + (1/15) \approx 7 + 0.067 = 7.067$
- Actual: $\sqrt{50} \approx 7.071$ (very close!)

Example: Estimate $\sqrt{120}$

Solution:

- Nearest perfect squares: 100 (10^2) and 121 (11^2)
- 120 is closer to 121 than to 100
- Difference: $121 - 120 = 1$, Total difference: $121 - 100 = 21$
- Estimate: $11 - (1/21) \approx 11 - 0.048 = 10.952$
- Actual: $\sqrt{120} \approx 10.954$ (very close!)

6. SSC CGL PRACTICE PROBLEMS

Previous Year Question Types

Problem 1: Find $\sqrt{1764}$

Solution:

- Prime factors: $2 \times 2 \times 3 \times 3 \times 7 \times 7$
- Group: $(2 \times 2) \times (3 \times 3) \times (7 \times 7)$
- $\sqrt{1764} = 2 \times 3 \times 7 = \mathbf{42}$

Problem 2: Simplify $\sqrt{50} + \sqrt{18} - \sqrt{8}$

Solution:

- $\sqrt{50} = \sqrt{(25 \times 2)} = 5\sqrt{2}$
- $\sqrt{18} = \sqrt{(9 \times 2)} = 3\sqrt{2}$
- $\sqrt{8} = \sqrt{(4 \times 2)} = 2\sqrt{2}$
- Expression = $5\sqrt{2} + 3\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$

Problem 3: If $\sqrt{x} = 12$, find x^2

Solution:

- $\sqrt{x} = 12 \Rightarrow x = 12^2 = 144$
- $x^2 = 144^2 = 20736$

Problem 4: Find the value of $\sqrt{0.0625}$

Solution:

- $0.0625 = 625/10000$
- $\sqrt{(625/10000)} = \sqrt{625}/\sqrt{10000} = 25/100 = 0.25$

7. QUICK METHODS & SHORTCUTS

Mental Calculation Tricks

For numbers ending with 5:

$(n5)^2 = n(n+1)$ followed by 25

Example: $35^2 = 3 \times 4 = 12$, add 25 $\rightarrow 1225$

Square Root Estimation:

- If number ends with 1, root ends with 1 or 9
- If number ends with 4, root ends with 2 or 8
- If number ends with 5, root ends with 5
- If number ends with 6, root ends with 4 or 6
- If number ends with 9, root ends with 3 or 7

Quick Perfect Square Check:

- Digital root should be 1,4,7,9
- Last two digits should be from specific patterns
- Even number of zeros at end

Important Square Roots to Memorize

Number	Square Root	Approximation
2	$\sqrt{2}$	1.414
3	$\sqrt{3}$	1.732
5	$\sqrt{5}$	2.236
6	$\sqrt{6}$	2.449
7	$\sqrt{7}$	2.646
8	$\sqrt{8}$	2.828
10	$\sqrt{10}$	3.162

8. SSC CGL PREPARATION STRATEGY

Expected Marks Distribution

Topic	Frequency	Difficulty	Marks Weightage
Perfect Square Roots	High	Easy	1-2 marks
Prime Factorization	Medium	Medium	1 mark
Long Division Method	Low	Hard	1 mark
Decimal/Fraction Roots	Medium	Medium	1-2 marks
Simplification	High	Medium	2-3 marks

15-Day Study Plan

Week 1: Foundation Building

- Days 1-3: Perfect squares and basic concepts
- Days 4-5: Prime factorization method
- Days 6-7: Long division method

Week 2: Advanced Topics & Practice

- Days 8-9: Decimal and fraction square roots
- Days 10-11: Approximation and estimation
- Days 12-13: Simplification problems
- Days 14-15: Previous year papers and revision

Daily Practice Routine:

- Memorize 5 new perfect squares daily
- Practice 10 prime factorization problems
- Solve 5 simplification problems
- Learn estimation techniques

Exam Tips

Time Management:

- Perfect squares: 30 seconds
- Prime factorization: 1-2 minutes

- Long division: 2-3 minutes
- Simplification: 1-2 minutes

Common Mistakes to Avoid:

- Forgetting negative roots when applicable
- Incorrect pairing in prime factorization
- Wrong decimal point placement
- Calculation errors in long division

Square Roots - SSC CGL Master Notes

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