

HEIGHTS AND DISTANCES

Complete SSC CGL Trigonometry Notes

Comprehensive Guide with Formulas & Examples • Created by GovtExamPrep

1. BASIC TRIGONOMETRIC CONCEPTS

Trigonometric Ratios

Trigonometric Ratios: Relationships between the angles and sides of a right-angled triangle.

[Diagram: Right-angled triangle showing opposite, adjacent, and hypotenuse sides]

Primary Trigonometric Ratios:

$\sin \theta = \text{Opposite/Hypotenuse}$

$\cos \theta = \text{Adjacent/Hypotenuse}$

$\tan \theta = \text{Opposite/Adjacent}$

Reciprocal Ratios:

$\operatorname{cosec} \theta = 1/\sin \theta$

$\sec \theta = 1/\cos \theta$

$\cot \theta = 1/\tan \theta$

Angle	sin	cos	tan
0°	0	1	0
30°	1/2	$\sqrt{3}/2$	1/√3
45°	1/√2	1/√2	1
60°	$\sqrt{3}/2$	1/2	√3
90°	1	0	∞

Angle of Elevation and Depression

Angle of Elevation: The angle between the horizontal line and the line of sight when looking upward at an object.

Angle of Depression: The angle between the horizontal line and the line of sight when looking downward at an object.

[Diagram: Showing angle of elevation and angle of depression]

Important: Angle of elevation = Angle of depression (alternate angles)

2. BASIC HEIGHT AND DISTANCE PROBLEMS

Single Object Problems

Example 1: The angle of elevation of the top of a tower from a point 20m away from its foot is 60° . Find the height of the tower.

Solution:

- Let height of tower = h
- Distance from tower = 20m
- Angle of elevation = 60°
- $\tan 60^\circ = h/20$
- $\sqrt{3} = h/20$
- $h = 20\sqrt{3} = \mathbf{34.64m}$

Example 2: A ladder leaning against a wall makes 30° with the ground. If ladder foot is 10m from wall, find ladder length.

Solution:

- Let ladder length = L
- Distance from wall = 10m
- $\cos 30^\circ = 10/L$
- $\sqrt{3}/2 = 10/L$
- $L = 20/\sqrt{3} = \mathbf{11.55m}$

Two Object Problems

Example: Two towers of heights 20m and 30m. Angle of elevation from midpoint between them are 45° and 60° respectively. Find distance between towers.

Solution:

- Let distance between towers = $2x$
- Distance to each tower from midpoint = x
- For first tower: $\tan 45^\circ = 20/x \Rightarrow 1 = 20/x \Rightarrow x = 20m$
- For second tower: $\tan 60^\circ = 30/x \Rightarrow \sqrt{3} = 30/x \Rightarrow x = 30/\sqrt{3} = 10\sqrt{3}m$

- Since x should be same, there's inconsistency in data
- **Note:** This shows importance of consistent data in problems

3. MOVING OBSERVER PROBLEMS

Approaching or Moving Away

Common Scenario: An observer moves towards or away from an object, and the angle of elevation changes.

Example: The angle of elevation of a tower from a point is 30° . On walking 20m towards the tower, it becomes 60° . Find tower height.

Solution:

- Let initial distance = x , height = h
- $\tan 30^\circ = h/x \Rightarrow 1/\sqrt{3} = h/x \Rightarrow x = h\sqrt{3} \dots(1)$
- After moving 20m closer: distance = $x - 20$
- $\tan 60^\circ = h/(x-20) \Rightarrow \sqrt{3} = h/(x-20) \Rightarrow x-20 = h/\sqrt{3} \dots(2)$
- From (1) and (2): $h\sqrt{3} - 20 = h/\sqrt{3}$
- $h\sqrt{3} - h/\sqrt{3} = 20$
- $h(\sqrt{3} - 1/\sqrt{3}) = 20$
- $h(3-1)/\sqrt{3} = 20$
- $2h/\sqrt{3} = 20$
- $h = 10\sqrt{3} = \mathbf{17.32m}$

Moving Perpendicularly

Example: From a point A, the angle of elevation of a tower is 30° . From point B, 20m perpendicular to AB, the angle is 45° . Find height.

Solution:

- Let height = h , distance from A = x
- From A: $\tan 30^\circ = h/x \Rightarrow x = h\sqrt{3}$
- From B: distance = $\sqrt{x^2 + 20^2}$
- $\tan 45^\circ = h/\sqrt{x^2 + 20^2} \Rightarrow 1 = h/\sqrt{x^2 + 20^2}$
- $h = \sqrt{x^2 + 20^2}$
- $h = \sqrt{3h^2 + 400}$
- $h^2 = 3h^2 + 400$
- $-2h^2 = 400$

- **Note:** This gives imaginary solution, indicating data inconsistency

4. SHADOW PROBLEMS

Sun's Altitude Problems

Key Concept: When sun's rays make an angle with horizontal, objects and their shadows form right triangles with same angle.

Example: The length of shadow of a tower is $\sqrt{3}$ times its height. Find sun's altitude.

Solution:

- Let height = h , shadow length = $\sqrt{3} h$
- $\tan \theta = \text{height/shadow} = h/(\sqrt{3} h) = 1/\sqrt{3}$
- $\theta = 30^\circ$
- Sun's altitude = **30°**

Example: If a pole 6m high casts shadow $2\sqrt{3}$ m long, find sun's elevation.

Solution:

- Height = 6m, Shadow = $2\sqrt{3}$ m
- $\tan \theta = 6/(2\sqrt{3}) = 3/\sqrt{3} = \sqrt{3}$
- $\theta = 60^\circ$
- Sun's elevation = **60°**

5. TWO OBJECTS - HEIGHT COMPARISON

Objects on Same Side

Example: Two towers of heights 20m and 30m. From midpoint between them, angles of elevation are complementary. Find distance between towers.

Solution:

- Let distance between towers = $2x$
- Distance to each from midpoint = x
- $\tan \theta = 20/x$
- $\tan (90^\circ - \theta) = 30/x \Rightarrow \cot \theta = 30/x$
- But $\cot \theta = 1/\tan \theta = x/20$
- Therefore, $x/20 = 30/x$
- $x^2 = 600$
- $x = 10\sqrt{6}$
- Distance between towers = $2x = \mathbf{20\sqrt{6} \text{ m}}$

Objects on Opposite Sides

Example: From top of 75m tower, angles of depression of two objects on opposite sides are 30° and 45° . Find distance between objects.

Solution:

- Tower height = 75m
- For first object: $\tan 30^\circ = 75/d_1 \Rightarrow d_1 = 75\sqrt{3}$
- For second object: $\tan 45^\circ = 75/d_2 \Rightarrow d_2 = 75$
- Distance between objects = $d_1 + d_2 = 75\sqrt{3} + 75$
- $= 75(\sqrt{3} + 1) = \mathbf{204.9m}$

6. RIVER AND BOAT PROBLEMS

Width of River Problems

Example: From top of 60m cliff, angle of depression of a boat is 30° . Find distance of boat from cliff.

Solution:

- Cliff height = 60m
- Angle of depression = 30°
- Angle of elevation from boat = 30°
- $\tan 30^\circ = 60/\text{distance}$
- $1/\sqrt{3} = 60/d$
- $d = 60\sqrt{3} = \mathbf{103.92m}$

Example: A man on cliff observes boat at 30° depression. After boat moves 40m towards cliff, depression becomes 60° . Find cliff height.

Solution:

- Let cliff height = h , initial distance = x
- $\tan 30^\circ = h/x \Rightarrow x = h\sqrt{3}$
- After moving 40m: distance = $x - 40$
- $\tan 60^\circ = h/(x-40) \Rightarrow x-40 = h/\sqrt{3}$
- $h\sqrt{3} - 40 = h/\sqrt{3}$
- $h\sqrt{3} - h/\sqrt{3} = 40$
- $h(3-1)/\sqrt{3} = 40$
- $2h/\sqrt{3} = 40$
- $h = 20\sqrt{3} = \mathbf{34.64m}$

7. SSC CGL PRACTICE PROBLEMS

Previous Year Question Types

Problem 1: A ladder makes 60° with ground when leaned against wall. If foot is 4m from wall, find ladder length.

Solution:

- $\cos 60^\circ = 4/\text{ladder length}$
- $1/2 = 4/L$
- $L = 8\text{m}$

Problem 2: From top of 30m tower, man observes car at 45° depression. Find car distance from tower.

Solution:

- $\tan 45^\circ = 30/\text{distance}$
- $1 = 30/d$
- $d = 30\text{m}$

Problem 3: Two poles 15m and 30m high. Line joining tops makes 45° with horizontal. Find distance between poles.

Solution:

- Height difference = $30 - 15 = 15\text{m}$
- $\tan 45^\circ = 15/\text{distance}$
- $1 = 15/d$
- $d = 15\text{m}$

8. QUICK FORMULAS & SHORTCUTS

Important Relationships

Standard Height-Distance Ratios:

- If angle = 30° : height : distance = $1 : \sqrt{3}$
- If angle = 45° : height : distance = $1 : 1$
- If angle = 60° : height : distance = $\sqrt{3} : 1$

Quick Calculation Formulas:

- Height = Distance $\times \tan \theta$
- Distance = Height $\times \cot \theta$
- Hypotenuse = Height $\times \operatorname{cosec} \theta$

Memory Tip: "Some People Have, Curly Black Hair, Through Proper Brushing"

Sin = Perp/Hyp, Cos = Base/Hyp, Tan = Perp/Base

Common Problem Patterns

Problem Type	Approach	Key Formula
Single object	Use basic trig ratio	$\tan \theta = \text{height/distance}$
Moving observer	Set up two equations	Solve simultaneous equations
Shadow problems	Use sun's angle	$\tan \theta = \text{height/shadow}$
Two objects	Use complementary angles	$\tan \theta \times \tan(90^\circ - \theta) = 1$

9. SSC CGL PREPARATION STRATEGY

Expected Marks Distribution

Topic	Frequency	Difficulty	Marks Weightage
Basic Problems	High	Easy	1-2 marks
Moving Observer	Medium	Medium	1-2 marks
Shadow Problems	Medium	Easy-Medium	1 mark
Two Objects	Low	Hard	1 mark
River Problems	Medium	Medium	1-2 marks

20-Day Study Plan

Week 1: Foundation Building

- Days 1-3: Trigonometric ratios and values
- Days 4-7: Basic height and distance problems
- Days 8-10: Angle of elevation and depression

Week 2-3: Advanced Practice

- Days 11-13: Moving observer problems
- Days 14-16: Shadow and two-object problems
- Days 17-20: Previous year papers and revision

Daily Practice: 10 basic problems, 5 moving observer problems, learn all trigonometric values, practice quick calculations.

Heights and Distances - SSC CGL Master Notes

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