CIRCLE & ITS CHORDS

Complete SSC CGL Master Notes

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1. CIRCLE FUNDAMENTALS

Basic Definition & Elements

A **circle** is a set of all points in a plane that are equidistant from a fixed point called the **center**.

Circle Elements:
• **Center (O)**: Fixed point
• **Radius (r)**: Distance from center to any point on circle
• **Diameter (d)**: Twice the radius (d = 2r)
• **Circumference**: Perimeter of the circle
• **Chord**: Line segment joining any two points on circle
• **Arc**: Part of circumference between two points

Basic Circle Formulas

Circumference: $C = 2\pi r = \pi d$
Area: $A = \pi r^2 = \pi d^2/4$
Diameter: $d = 2r$
Radius: $r = d/2 = C/(2\pi)$

2. CHORDS - DEFINITION & PROPERTIES

What is a Chord?

A **chord** is a line segment whose endpoints lie on the circle.
The **diameter** is the longest chord of a circle, passing through the center.

Chord Length Formula:
Length of chord = $2\sqrt{(r^2 - d^2)}$
where r = radius, d = perpendicular distance from center to chord

Important Chord Properties

Property	Description	Formula
Equal Chords	Chords equidistant from center are equal	If $d_1 = d_2$, then $chord_1 = chord_2$
Perpendicular from Center	Line from center perpendicular to chord bisects it	If $OM \perp AB$, then $AM = MB$
Equal Arcs	Equal chords subtend equal arcs	If $chord_1 = chord_2$, then $arc_1 = arc_2$

3. CHORD LENGTH CALCULATIONS

Chord Length Formulas

Method 1: Using radius and distance from center
Chord length = $2\sqrt{(r^2 - d^2)}$

Method 2: Using radius and central angle θ (in radians)
Chord length = $2r \times \sin(\theta/2)$

Method 3: Using radius and central angle θ (in degrees)
Chord length = $2r \times \sin(\theta^\circ \times \pi/360)$

Practice Problems

Example 1: Circle radius 10cm, chord distance from center 6cm. Find chord length.

Solution:

- Chord length = $2\sqrt{(r^2 - d^2)}$
- = $2\sqrt{(10^2 - 6^2)} = 2\sqrt{(100 - 36)}$
- = $2\sqrt{64} = 2\times8 =$ **16 cm**

Example 2: Circle radius 14cm, central angle 60°. Find chord length.

Solution:

- Chord length = $2r \times \sin(\theta/2)$
- = $2\times14 \times \sin(60^\circ/2) = 28 \times \sin(30^\circ)$
- = $28 \times 0.5 =$ **14 cm**

4. THEOREMS RELATED TO CHORDS

Theorem 1: Perpendicular from Center

Theorem: The perpendicular from the center of a circle to a chord bisects the chord.
Converse: The line joining the center of a circle to the midpoint of a chord is perpendicular to the chord.

Proof:

In circle with center O, chord AB, OM ⊥ AB

- OA = OB (radii)
- OM common
- ∠OMA = ∠OMB = 90°
- ΔOAM ≅ ΔOBM (RHS congruence)
- Therefore, AM = MB

Theorem 2: Equal Chords Equidistant

Theorem: Equal chords of a circle are equidistant from the center.
Converse: Chords equidistant from the center are equal.

5. INTERSECTING CHORDS THEOREM

Theorem Statement

Intersecting Chords Theorem:

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When two chords intersect inside a circle, the product
of the segments of one chord equals the product of the
segments of the other chord.

If chords AB and CD intersect at point P, then:
AP × PB = CP × PD
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Applications & Examples

Example: Two chords intersect. Segments are 4cm, 6cm and 3cm, x cm. Find x.

Solution:

- Using theorem: $4 \times 6 = 3 \times x$
- $24 = 3x$
- $x = 24/3 =$ **8 cm**

6. TANGENTS & SECANTS

Tangent Properties

Tangent-Radius Theorem:
The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Tangent Length Theorem:
Tangents from an external point to a circle are equal in length.

Secant-Tangent Theorem

Secant-Tangent Theorem:
If a tangent and secant are drawn from an external point, then:
$(\text{Tangent})^2 = (\text{External segment of secant}) \times (\text{Whole secant})$

$PT^2 = PA \times PB$

7. CYCLIC QUADRILATERALS

Properties

Cyclic Quadrilateral Properties:

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1. Sum of opposite angles = 180°
2. Exterior angle = Interior opposite angle
3. Ptolemy's Theorem: AC × BD = AB × CD + AD × BC
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Ptolemy's Theorem Applications

Example: Cyclic quadrilateral sides 5,6,7,8. Find product of diagonals.

Solution:

- Using Ptolemy: $AC \times BD = AB \times CD + AD \times BC$
- $= 5\times7 + 6\times8 = 35 + 48 =$ **83**

8. SSC CGL PRACTICE PROBLEMS

Problem Set with Solutions

Problem 1: Circle radius 25cm, chord length 48cm. Find distance from center.

Solution:

- Using chord formula: chord = $2\sqrt{(r^2 - d^2)}$
- $48 = 2\sqrt{(625 - d^2)}$
- $24 = \sqrt{(625 - d^2)}$
- $576 = 625 - d^2$
- $d^2 = 49 \Rightarrow d =$ **7 cm**

Problem 2: Two chords intersect. Segments 3cm, 4cm and 2cm, x cm. Find x.

Solution:

- Using intersecting chords: $3 \times 4 = 2 \times x$
- $12 = 2x \Rightarrow x =$ **6 cm**

Problem 3: Equal chords of length 24cm in circle radius 15cm. Find distance between chords.

Solution:

- Distance from center to one chord: $d = \sqrt{(15^2 - 12^2)} = \sqrt{(225-144)} = \sqrt{81} = 9$cm
- If chords on same side: distance = 0
- If chords on opposite sides: distance = 9 + 9 = **18 cm**

Important Circle Theorems for SSC CGL

Must-Know Theorems:
1. Angle in semicircle = 90°
2. Angle at center = 2 × angle at circumference
3. Angles in same segment are equal
4. Opposite angles of cyclic quadrilateral = 180°

5. Tangent ⊥ Radius
6. Tangents from external point are equal
7. Intersecting chords theorem
8. Alternate segment theorem

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