COMPLETE ALGEBRA FORMULAS & TRICKS

SSC CGL Master Notes - All Essential Formulas with Memory Techniques

Identities, Equations, Progressions, Surds, Indices & Shortcut Methods

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1. BASIC ALGEBRAIC IDENTITIES

Square Formulas

$(a + b)^2$
$= a^2 + 2ab + b^2$

$(a - b)^2$
$= a^2 - 2ab + b^2$

$a^2 - b^2$
$= (a + b)(a - b)$

Memory Tip: For $(a + b)^2$, remember "Square the first, square the second, twice the product" → $a^2 + b^2 + 2ab$. For $(a - b)^2$, it's "Square the first, square the second, minus twice the product" → $a^2 + b^2 - 2ab$.

Cube Formulas

$(a + b)^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$
$= a^3 + b^3 + 3ab(a + b)$

$(a - b)^3$
$= a^3 - 3a^2b + 3ab^2 - b^3$
$= a^3 - b^3 - 3ab(a - b)$

$a^3 + b^3$
$= (a + b)(a^2 - ab + b^2)$

$a^3 - b^3$

$= (a - b)(a^2 + ab + b^2)$

Quick Trick: For $(a + b)^3$, use the pattern: "Cube the first, three times square first into second, three times first into square second, cube the second" → $a^3 + 3a^2b + 3ab^2 + b^3$.

2. SPECIAL IDENTITIES

Advanced Identities

- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $a^2 + b^2 + c^2 - ab - bc - ca = ½[(a-b)^2 + (b-c)^2 + (c-a)^2]$
- $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
- If $a+b+c=0$ then $a^3+b^3+c^3=3abc$

Example: If $a+b+c=0$, prove that $a^3+b^3+c^3=3abc$

Solution:

We know: $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Given: $a+b+c = 0$

Therefore: $a^3+b^3+c^3-3abc = 0 \times (a^2+b^2+c^2-ab-bc-ca) = 0$

Hence: $a^3+b^3+c^3 = 3abc$

Useful Shortcut Identities

$(a + b)^2 + (a - b)^2$
$= 2(a^2 + b^2)$

$(a + b)^2 - (a - b)^2$
$= 4ab$

$(a + b)^4 - (a - b)^4$
$= 8ab(a^2 + b^2)$

3. FACTORIZATION FORMULAS

Common Factorizations

- $a^2 - b^2 = (a + b)(a - b)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a - b)(a + b)(a^2 + b^2)$
- $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

Quadratic Factorization

For $ax^2 + bx + c = 0$:
Find factors of ac that add to b
Example: $6x^2 + 11x + 3$
ac = 18, factors 9 and 2 add to 11
$6x^2 + 9x + 2x + 3 = 3x(2x+3) + 1(2x+3) = (3x+1)(2x+3)$

Memory Tip: For quadratic factorization, remember the acronym "FOIL" - First, Outer, Inner, Last. This helps in both factorization and expansion.

4. LINEAR EQUATIONS

Solving Methods

- $ax + b = 0 \Rightarrow x = -b/a$
- $ax + b = cx + d \Rightarrow x = (d - b)/(a - c)$
- System: $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$

Cross Multiplication Method

For:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$$x/(b_1c_2 - b_2c_1) = y/(c_1a_2 - c_2a_1) = 1/(a_1b_2 - a_2b_1)$$

Therefore:

$x = (b_1c_2 - b_2c_1)/(a_1b_2 - a_2b_1)$

$y = (c_1a_2 - c_2a_1)/(a_1b_2 - a_2b_1)$

Quick Trick: For cross multiplication, write coefficients in this pattern:
a_1 b_1 c_1 a_1 b_1
a_2 b_2 c_2 a_2 b_2
Then $x = (b_1c_2 - b_2c_1)/D$, $y = (c_1a_2 - c_2a_1)/D$ where $D = a_1b_2 - a_2b_1$

5. QUADRATIC EQUATIONS

Standard Form & Solutions

```
ax² + bx + c = 0 (where a ≠ 0)

• Roots: x = [-b ± √(b² - 4ac)]/(2a)
• Discriminant: D = b² - 4ac
• Nature of roots:
  - D > 0: Real and distinct
  - D = 0: Real and equal
  - D < 0: Imaginary
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Relations between Roots and Coefficients

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For ax² + bx + c = 0 with roots α and β:

• Sum of roots: α + β = -b/a
• Product of roots: αβ = c/a
• Difference of roots: |α - β| = √D/|a|
• α² + β² = (α+β)² - 2αβ = (b²-2ac)/a²
• α³ + β³ = (α+β)³ - 3αβ(α+β) = (-b³+3abc)/a³
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Memory Tip: For sum and product of roots, remember "Sum is -b/a, Product is c/a" - the signs alternate and follow the equation order.

Forming Quadratic Equations

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If roots are α and β, then:
x² - (sum of roots)x + (product of roots) = 0
x² - (α+β)x + αβ = 0
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Example: Form quadratic equation with roots 2 and 3

Solution:

Sum of roots = 2 + 3 = 5

Product of roots = 2 × 3 = 6

Equation: $x^2 - 5x + 6 = 0$

6. PROGRESSIONS

Arithmetic Progression (AP)

- nth term: $a_n = a + (n-1)d$
- Sum of n terms: $S_n = n/2[2a + (n-1)d] = n/2(a + l)$
- Common difference: $d = a_n - a_{n-1}$
- Number of terms: $n = [(l-a)/d] + 1$

Quick Trick: For finding middle term in odd number of terms in AP, if n is odd, middle term = (n+1)/2 th term. Average of AP = (first term + last term)/2.

Geometric Progression (GP)

- nth term: $a_n = ar^{n-1}$
- Sum of n terms: $S_n = a(1-r^n)/(1-r)$ for $r \neq 1$
- Sum to infinity: $S_\infty = a/(1-r)$ for $|r|<1$
- Common ratio: $r = a_n/a_{n-1}$

Harmonic Progression (HP)

- Sequence where reciprocals form AP
- nth term of HP = $1/[a + (n-1)d]$
- No simple sum formula for HP
- Harmonic Mean between a and b = $2ab/(a+b)$

Memory Tip: Remember the progression types: AP - Add/Subtract constantly, GP - Multiply/Divide constantly, HP - Reciprocals form AP.

7. SURDS & INDICES

Laws of Indices

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^{-n} = 1/a^n$
- $a^0 = 1$ $(a \neq 0)$
- $a^{1/n} = \sqrt[n]{a}$
- $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Laws of Surds

- $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$
- $\sqrt{a} \div \sqrt{b} = \sqrt{(a/b)}$
- $(\sqrt[n]{a})^n = a$
- $\sqrt[n]{(ab)} = \sqrt[n]{a} \times \sqrt[n]{b}$
- $\sqrt[n]{(a/b)} = \sqrt[n]{a} \div \sqrt[n]{b}$
- $\sqrt[m]{(\sqrt[n]{a})} = \sqrt[mn]{a}$

Quick Trick: To rationalize denominator with $\sqrt{a}$, multiply numerator and denominator by $\sqrt{a}$. For $a+\sqrt{b}$, multiply by $a-\sqrt{b}$ (conjugate).

Comparison of Surds

To compare $\sqrt[n]{a}$ and $\sqrt[m]{b}$:
- Raise both to power LCM(n,m)
- Example: Compare $\sqrt[3]{4}$ and $\sqrt{2}$
- LCM(3,2)=6, so compare $(\sqrt[3]{4})^6=4^2=16$ and $(\sqrt{2})^6=2^3=8$
- Therefore $\sqrt[3]{4} > \sqrt{2}$

8. LOGARITHMS

Definition & Basic Laws

If $a^x = N$ then $\log_a N = x$ $(a>0, a\neq1, N>0)$

- $\log_a(mn) = \log_a m + \log_a n$
- $\log_a(m/n) = \log_a m - \log_a n$
- $\log_a m^n = n \log_a m$
- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a b = 1/\log_b a$ (Change of base)
- $a^{\log_a N} = N$

Special Cases & Tricks

- $\log(1+x) \approx x$ for small x
- $\log_a b \times \log_b c \times \log_c a = 1$
- $\log_a b = \log b / \log a$ (Common log)
- $\log_{10}10 = 1$, $\log_{10}100 = 2$, etc.
- $\log_2 \approx 0.3010$, $\log_3 \approx 0.4771$, $\log_5 \approx 0.6990$

Memory Tip: For logarithm laws, remember they're opposite of exponent laws: Multiplication becomes addition, Division becomes subtraction, Power becomes multiplication.

9. SETS & FUNCTIONS

Set Operations

- $A \cup B = \{x: x \in A \text{ or } x \in B\}$ (Union)
- $A \cap B = \{x: x \in A \text{ and } x \in B\}$ (Intersection)
- $A - B = \{x: x \in A \text{ and } x \notin B\}$ (Difference)
- $A' = \{x: x \notin A\}$ (Complement)
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A)+n(B)+n(C)-n(A \cap B)-n(B \cap C)-n(C \cap A)+n(A \cap B \cap C)$

Function Properties

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x) \times g(x)$
- $(f/g)(x) = f(x)/g(x)$, $g(x) \neq 0$
- $(f \circ g)(x) = f(g(x))$ (Composition)
- If $f(a)=b$ then $f^{-1}(b)=a$ (Inverse)

10. QUICK CALCULATION TRICKS

Mental Math Shortcuts

Multiplication by 11:
- 23 × 11 = 2 (2+3) 3 = 253
- 57 × 11 = 5 (5+7) 7 = 5 12 7 = 627 (carry over)

Squares ending with 5:
- $(a5)^2 = a \times (a+1)$ followed by 25
- $25^2 = 2 \times 3 = 6$, so 625
- $75^2 = 7 \times 8 = 56$, so 5625

Percentage Calculations:
- x% of y = y% of x
- 25% = 1/4, 50% = 1/2, 75% = 3/4
- 12.5% = 1/8, 37.5% = 3/8, 62.5% = 5/8, 87.5% = 7/8

Algebra Shortcuts

Finding (a+b) when a^2+b^2 and ab are given:
- $(a+b)^2 = a^2+b^2+2ab$
- So $a+b = \sqrt{(a^2+b^2+2ab)}$

Finding (a-b) when a^2+b^2 and ab are given:
- $(a-b)^2 = a^2+b^2-2ab$
- So $a-b = \sqrt{(a^2+b^2-2ab)}$

Finding a^3+b^3 when a+b and ab are given:
- $a^3+b^3 = (a+b)^3 - 3ab(a+b)$

Equation Solving Tricks

For ax=b type equations:
- If a and b have same signs, x is positive
- If a and b have opposite signs, x is negative

For quadratic equations:
• If sum of coefficients=0, then x=1 is a root
• If sum of coefficients with alternate signs=0, then x=-1 is a root

For simultaneous equations:
• If $a_1/a_2 = b_1/b_2 = c_1/c_2$, infinite solutions
• If $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, no solution
• Otherwise, unique solution

Exam Strategy

Time-Saving Tips for SSC CGL:
1. Memorize squares up to 30 and cubes up to 15
2. Learn common algebraic identities by heart
3. Practice mental calculation for basic operations
4. Use elimination method in MCQs
5. Check units and dimensions in answers
6. Verify if answer seems reasonable
7. For tough problems, try substitution method
8. Remember special cases and shortcuts

Final Revision Strategy: Create formula cards with identity on one side and example on the other. Practice 10 problems daily from each topic. Focus on understanding concepts rather than rote memorization. For SSC CGL, pay special attention to identities, quadratic equations, and progressions as they carry maximum weightage.

Practice regularly and master these formulas for success in the quantitative aptitude section.