AVERAGE, RATIO & PROPORTION

Master SSC CGL Examination Notes

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1. AVERAGE - COMPLETE MASTERY

Basic Definition & Formula

Average is the mean value which represents the central value of a set of numbers. It's calculated by dividing the sum of all values by the number of values.

Basic Average Formula:

$$\text{Average} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Derived Formulas:

$$\text{Sum of observations} = \text{Average} \times \text{Number of observations}$$

$$\text{Number of observations} = \frac{\text{Sum of observations}}{\text{Average}}$$

Weighted Average Concept

Weighted Average Formula:

$$\text{Weighted Average} = \frac{w_1x_1 + w_2x_2 + \ldots + w_nx_n}{w_1 + w_2 + \ldots + w_n}$$

Where:

$w_1, w_2, \ldots, w_n$ = Weights of different groups

$x_1, x_2, \ldots, x_n$ = Averages of different groups

Example: Class A has 30 students with average 60, Class B has 20 students with average 80

Solution:

- Weighted Average = (30×60 + 20×80) / (30+20)
- = (1800 + 1600) / 50
- = 3400 / 50 = **68**

2. ADVANCED AVERAGE PROBLEM TYPES

Type 1: Inclusion/Exclusion Problems

Problem: Average of 25 numbers is 18. If 2 numbers 35 and 40 are included, new average?

Detailed Solution:

- Sum of 25 numbers = 25 × 18 = 450
- New sum after inclusion = 450 + 35 + 40 = 525
- New count = 25 + 2 = 27
- New average = 525 / 27 = **19.44**

Type 2: Replacement Problems

Problem: Average of 30 numbers is 15. If one number 25 is replaced by 35, new average?

Detailed Solution:

- Total change = New value - Old value = 35 - 25 = +10
- Change in average = Total change / Number of terms = 10/30 = 0.333
- New average = 15 + 0.333 = **15.333**

Type 3: Age Average Problems

Problem: Average age of 20 students is 15 years. Teacher's age 35 years included

Detailed Solution:

- Sum of students' age = 20 × 15 = 300 years
- New total age = 300 + 35 = 335 years
- New average = 335 / 21 = **15.95 years**

3. RATIO & PROPORTION - FOUNDATION

Basic Concepts

Ratio: Comparison of two quantities by division. a:b = a/b
Proportion: Equality of two ratios. a:b = c:d or a/b = c/d
Continued Proportion: a:b = b:c (b is mean proportional)

If a:b = c:d, then:

- `a/b = c/d (Basic proportion)`
- `b/a = d/c (Invertendo)`
- `a/c = b/d (Alternendo)`
- `(a+b)/b = (c+d)/d (Componendo)`
- `(a-b)/b = (c-d)/d (Dividendo)`
- `(a+b)/(a-b) = (c+d)/(c-d) (Componendo-Dividendo)`

Mean Proportional

Mean Proportional between a and c:

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b = √(a × c)
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Third Proportional to a and b:

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c = (b × b) / a
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Fourth Proportional to a, b, c:

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d = (b × c) / a
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4. PARTNERSHIP & MIXTURE APPLICATIONS

Partnership Formulas

Profit Sharing Ratio:

$$\text{Profit Share} \propto (\text{Investment} \times \text{Time})$$

If A invests ₹X for T_1 months and B invests ₹Y for T_2 months:

$$\text{Profit Ratio} = (X \times T_1) : (Y \times T_2)$$

For multiple partners:

$$\text{Profit Ratio} = (\text{Capital}_1 \times \text{Time}_1) : (\text{Capital}_2 \times \text{Time}_2) : \ldots : (\text{Capital}_n \times \text{Time}_n)$$

Problem: A invests ₹6000 for 4 months, B invests ₹8000 for 6 months, C invests ₹10000 for 3 months

Solution:

- Profit Ratio = (6000×4) : (8000×6) : (10000×3)
- = 24000 : 48000 : 30000
- Simplify by dividing by 6000 = 4 : 8 : 5
- **Profit Ratio = 4:8:5**

Mixture & Alligation

Alligation Rule:
Quantity of Cheaper / Quantity of Dearer = (Price of Dearer - Mean Price) / (Mean Price - Price of Cheaper)

Graphical Representation:
Dearer Price --- (Mean Price - Cheaper Price)
Mean Price
Cheaper Price --- (Dearer Price - Mean Price)

Problem: Mix rice costing ₹20/kg and ₹30/kg to get mixture at ₹25/kg in ratio?

Solution using Alligation:

- Cheaper rice (₹20) --- (30-25) = 5
- Mean Price (₹25)
- Dearer rice (₹30) --- (25-20) = 5
- Ratio = 5:5 = **1:1**

5. ADVANCED RATIO PROBLEMS

Type 1: Ratio Change Problems

Problem: Two numbers are in ratio 3:4. If each number increased by 5, ratio becomes 4:5

Detailed Solution:

- Let numbers be 3x and 4x
- After increase: (3x+5)/(4x+5) = 4/5
- Cross multiply: 5(3x+5) = 4(4x+5)
- 15x + 25 = 16x + 20
- 16x - 15x = 25 - 20
- x = 5
- Numbers: 3×5 = **15** and 4×5 = **20**

Type 2: Income-Expenditure-Savings

Problem: Income ratio A:B = 3:4, Expenditure ratio A:B = 2:3. Both save ₹1000 each

Detailed Solution:

- Let incomes: 3x, 4x
- Let expenditures: 2y, 3y
- Savings: 3x - 2y = 1000 ...(1)
- 4x - 3y = 1000 ...(2)
- Multiply (1) by 3: 9x - 6y = 3000
- Multiply (2) by 2: 8x - 6y = 2000
- Subtract: (9x-6y) - (8x-6y) = 3000-2000
- x = 1000
- From (1): 3(1000) - 2y = 1000 → 3000 - 2y = 1000
- 2y = 2000 → y = 1000
- Incomes: A = ₹3000, B = ₹4000

6. COMPREHENSIVE PRACTICE SET

Problem 1: Average of 11 results is 50. Average of first 6 is 49, last 6 is 52. Find 6th number

Detailed Solution:

- Sum of all 11 numbers = 11 × 50 = 550
- Sum of first 6 numbers = 6 × 49 = 294
- Sum of last 6 numbers = 6 × 52 = 312
- Sum of first 6 + last 6 = 294 + 312 = 606
- 6th number is counted twice in this sum
- 6th number = 606 - 550 = **56**

Problem 2: If a:b = 3:4, b:c = 5:6, find a:b:c

Detailed Solution:

- Make b same in both ratios
- a:b = 3:4 = 15:20 (multiply by 5)
- b:c = 5:6 = 20:24 (multiply by 4)
- Now b is 20 in both ratios
- a:b:c = **15:20:24**

Problem 3: In mixture of 60L, milk:water = 2:1. How much water to add to make ratio 1:2?

Detailed Solution:

- Current mixture: Milk = (2/3)×60 = 40L, Water = 20L
- Let x liters water added
- New ratio: 40/(20+x) = 1/2
- Cross multiply: 80 = 20 + x
- x = 80 - 20 = **60 liters**

7. SHORTCUTS & QUICK METHODS

Average Shortcuts

Shortcut 1: If all numbers increased by x, average increases by x
Shortcut 2: If all numbers multiplied by x, average multiplied by x
Shortcut 3: Average of consecutive numbers = (First + Last)/2
Shortcut 4: Average of consecutive even/odd numbers = Middle term

Ratio Shortcuts

Shortcut 1: If a:b = m:n, then a/(a+b) = m/(m+n)
Shortcut 2: If a/b = c/d, then (a+b)/(a-b) = (c+d)/(c-d)
Shortcut 3: To find a:b:c from a:b and b:c, take LCM of b's values
Shortcut 4: In mixture problems, use alligation rule directly

Memory Table - Common Ratios

Situation	Typical Ratio Pattern	Application
Partnership	Investment × Time	Profit sharing
Mixture	Alligation rule	Finding quantities
Age problems	Difference constant	Future/past ratios
Income-Expense	I_1-E_1 = I_2-E_2	Savings equal

8. SSC CGL PREVIOUS YEAR PATTERNS

Common Question Types

Type 1: Average of numbers with inclusion/exclusion
Type 2: Ratio change after addition/subtraction
Type 3: Partnership profit sharing
Type 4: Mixture alligation problems
Type 5: Age-based ratio problems
Type 6: Income-expenditure-savings ratio

Expected Marks Distribution

Topic	Frequency	Difficulty	Marks Weightage
Average	High	Medium	3-4 marks
Ratio	High	Medium	3-4 marks
Proportion	Medium	Easy	1-2 marks
Partnership	Medium	Medium	2-3 marks
Mixture	Medium	Hard	2-3 marks

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